

Enrollment No: _____

Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Engineering Mathematics-I

Subject Code: 4TE01EMT2

Branch: B.Tech(All)

Semester: 1

Date: 22/03/2017

Time: 10:30 to 01:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions:**(14)**

- a) If $z = 1 + \sqrt{3}i$ then $|\bar{z}| =$ _____.
- (a) 1 (b) 2 (c) $\sqrt{3}$ (d) $1 - \sqrt{3}i$
- b) Principal argument of $z = i + 1$ is _____.
- (a) $e^{\frac{3\pi}{4}i}$ (b) $\sqrt{2}$ (c) $e^{\frac{\pi}{4}i}$ (d) $e^{-\frac{\pi}{4}i}$
- c) $e^{\frac{\pi}{2}i} =$ _____.
- (a) 1 (b) -1 (c) i (d) $-i$
- d) Find n^{th} derivative of 3^x .
- e) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n =$ _____.
- (a) 1 (b) e (c) $\log 1$ (d) $\frac{1}{e}$
- f) $\lim_{x \rightarrow 0} \frac{x}{\tan 3x} =$ _____.
- (a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0
- g) n^{th} derivative of $y = \log(3 - 2x)$ is
- (a) $\frac{2^n n!}{(3 - 2x)^{n+1}}$ (b) $\frac{(-1)^n (-2)^n n!}{(3 - 2x)^n}$ (c) $\frac{-(2)^n (n-1)!}{(3 - 2x)^n}$ (d) $\frac{(-1)^{n-1} (-2)^n (n-1)!}{(3 - 2x)^{n+1}}$



- h) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ represent expansion of
 (a) $\sin x$ (b) $\cos x$ (c) $\sinh x$ (d) $\cosh x$
- i) If $y = \cos^{-1} x$ then x equal to
 (a) $1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \dots$ (b) $y - \frac{y^3}{3!} + \frac{y^5}{5!} - \dots$ (c) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots$ (d) none of these
- j) State Euler's theorem for homogeneous function of two variables x and y .
- k) Find $\frac{dy}{dx}$ if $x^2 + y^2 + 1 = 0$
- l) What is the value of $\frac{\partial}{\partial y}(y^x) = \underline{\hspace{2cm}}$.
 (a) y^x (b) $y^x \log y$ (c) xy^{x-1} (d) x^y
- m) A square matrix A is called orthogonal if
 (a) $AA^{-1} = I$ (b) $A^2 = A$ (c) $AA^T = I$ (d) $A^2 = I$
- n) The rank of the non-zero scalar matrix of order 3 is $\underline{\hspace{2cm}}$.
 (a) 1 (b) 2 (c) 3 (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

- a) Find the cube root of unity and prove that the sum of the roots is 0. (05)
- b) Find the real and imaginary part of $i^{\ln(1+i)}$. (05)
- c) Simplify: $\frac{(\cos 2\theta + i \sin 2\theta)^6 (\cos \theta - i \sin \theta)^{15}}{(\cos 3\theta - i \sin 3\theta)^2 (\cos 5\theta - i \sin 5\theta)^{-2}}$ (04)

Q-3 Attempt all questions

- a) If $y = e^{m \cos^{-1} x}$ then show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ (05)
- b) Find the n^{th} derivative of $y = \frac{x}{x^2 + a^2}$. (05)
- c) Expand $e^x \sin x$ in powers of x by Maclaurin's theorem up to the containing x^5 . (04)

Q-4 Attempt all questions

- a) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x} \right)$ (05)
- b) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{1 - \cos x}$ (05)
- c) Expand $f(x) = x^4 - 11x^3 + 43x^2 - 60x + 14$ in powers of $(x - 3)$. (04)



Q-5 Attempt all questions (14)

a) Solve the linear equation $\frac{1}{x} + \frac{1}{y} + \frac{2}{z} = 8$; $-\frac{1}{x} - \frac{2}{y} + \frac{3}{z} = 1$; $\frac{3}{x} - \frac{7}{y} + \frac{4}{z} = 10$. (05)

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ by normal form. (05)

c) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + e^x - 3}{2x} \right)$ (04)

Q-6 Attempt all questions

a) If $u = x^2 \tan^{-1} \left(\frac{y}{x} \right)$ then find $\frac{\partial^2 u}{\partial x \partial y}$. (05)

b) If $x = r \cos \theta$, $y = r \sin \theta$ then prove that $\left(\frac{\partial r}{\partial x} \right)_y = \left(\frac{\partial x}{\partial r} \right)_\theta$ & $\left(\frac{\partial \theta}{\partial x} \right)_y = \frac{1}{r^2} \left(\frac{\partial x}{\partial \theta} \right)_r$. (05)

c) If $x = r \cos \theta$, $y = r \sin \theta$ then prove that $J \cdot J' = 1$. (04)

Q-7 Attempt all questions

a) i) Define: Homogeneous function (07)

ii) Prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \sin u \cos 3u$ if $u = \tan^{-1} (x^2 + 2y^2)$.

b) Find the Eigen values and Eigenvectors for the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$. (07)

Q-8 Attempt all questions

a) Find the maxima and minima of $xy + 27 \left(\frac{1}{x} + \frac{1}{y} \right)$. (05)

b) Check the consistency and if consistent, solve the following system of equations (05)
 $3x + 3y + 2z = 1$; $x + 2y = 4$; $10y + 3z = -2$; $2x - 3y - z = 5$

c) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 1 & -1 \\ 5 & 0 & 1 \end{bmatrix}$ by Gauss Jordan method. (04)

